Sesión Especial 18

Loci of Riemann and Klein Surfaces with Automorphisms

Organizadores

- Antonio F. Costa (UNED)
- Milagros Izquierdo (Universidad de Linköping)

Descripción

The special session will be devoted to Riemann and Klein surfaces and their moduli spaces, with special attention to group actions on these types of surfaces, automorphisms of surfaces (real and complex), Grothendieck theory of dessins d'enfants (maps and hypermaps) and topological properties of moduli spaces of complex and real curves.

The study of Riemann surfaces with automorphisms constitutes an important meeting point for Group Theory, Geometry and Analysis, and there is considerable current activity in this field.

The special session will have a computational/combinatorial flavor, with focus on group actions on Riemann surfaces, Klein surfaces and related structures such as abelian varieties or hyperbolic manifolds.

Among others, the following topics will be covered:

Real and Complex Algebraic Curves and Surfaces

Automorphisms of Riemann and Klein Surfaces.

Dessins d'Enfants, Combinatorics and Graph Theory.

Theichmüller Theory and Moduli Spaces of Algebraic Curves

Proyecto MTM2017-90897-REDT

Programa

Lunes, 4 de febrero (mañana)

11:30 – 12:00	Antonio F. Costa (Universidad Nacional de Educación a Distancia) *One-dimensional families of Riemann surfaces of genus g with 4g+4 automorphisms*
12:00 – 12:30	Sebastián Reyes-Carocca (Universidad de La Frontera) *On large automorphism groups of compact Riemann surfaces*
12:30 – 13:00	Gabino González (Universidad Autónoma de Madrid) *Galois action on solenoids*
13:00 – 13:30	T. Shaska (Oackland University) *Superelliptic curves with complex multiplication*

Lunes, 4 de febrero (tarde)

17:00 – 17:30	Raquel Díaz (Universidad Complutense de Madrid) *Limit points of the hyperelliptic locus*
17:30 – 18:00	Saúl Quispe (Universidad de la Frontera) *Riemann surfaces defined over the reals*
18:00 – 18:30	Anita M. Rojas (Universidad de Chile) *Decomposition of Jacobian varieties of curves with dihedral actions via equisymmetric stratification*
18:30 – 19:00	Milagros Izquierdo (Linköping University) *On the Connectedness of the Branch Locus in Schottky Space*

Martes, 5 de febrero (mañana)

11:30 – 12:00	Ewa Kozłowska-Walania (Gdańsk University) *Properties of extremal Riemann surfaces*
12:00 – 12:30	Adrián Bacelo (Universidad Rey Juan Carlos) *The symmetric crosscap spectrum of Abelian groups*
12:30 – 13:00	Javier Cirre (Universidad Nacional de Educación a Distancia) *Largest automorphism groups of pseudo-real Riemann surfaces*
13:00 – 13:30	Ewa Tyszkowska (Gdańsk University) *Fixed points on asymmetric surfaces*

The symmetric crosscap spectrum of Abelian groups

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Abstract. The non-orientable non-bordered Klein surfaces, also named non-orientable Riemann surfaces, are compact, non-bordered, non-orientable surfaces endowed with a dianalytic structure. Every finite group G acts as an automorphism group of some of these surfaces. The minimal topological genus of them is called the symmetric crosscap number of the group.

In the last years, a great improvement in this parameter has been done, although it is the least known of some similar ones. In this oral communication we will talk about the symmetric crosscap spectrum of abelian groups, that is, we study which natural numbers are symmetric crosscap number of some abelian group. We will give some interesting results and fill some gaps of the symmetric crosscap spectrum.

Joint work with José Javier Etayo and Ernesto Martínez
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Largest automorphism groups of pseudo-real Riemann surfaces

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Abstract. A compact Riemann surface is said to be *pseudo-real* if it admits anti-conformal automorphisms but no such automorphisms of order 2. Any such surface lies in the so-called real moduli of the moduli space of compact Riemann surfaces, but cannot be defined by real polynomials. Observe that, from its very definition, assuring that a given Riemann surface is pseudo-real requires the knowledge of its full automorphism group.
An important aspect of current research on pseudo-real surfaces has to do with finding the largest groups of a prescribed type acting on them. Cyclic groups are the first to be considered. We will see that the largest order of an anti-conformal automorphism acting on a pseudo-real surface of genus g is $2g$ if g is even and $2g-2$ if g is odd, the bounds being attained for every g. Other types of groups can also be considered, as abelian groups. If no restriction on the type of group is imposed then the largest order, denoted by $M(g)$, satisfies $M(g) \leq 12(g-1)$, the bound being attained for infinitely many values of g. On the other hand, relatively little is known about lower bounds for $M(g)$. Accordingly, several questions regarding potential lower bounds for $M(g)$ are being considered. In this talk we will report on the current research concerning these questions and other related topics.

Joint work in progress with E. Bujalance and M. D. E. Conder.
Partially supported by MTM2014-55812.

One-dimensional families of Riemann surfaces of genus g with 4g+4 automorphisms

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Abstract. We will see that he maximal number $ag+b$ of automorphisms of equisymmetric and uniparametric families of Riemann surfaces appearing in all genera is $4g+4$. For all genera $g$greater than 1 there is an equisymmetric, uniparametric family $\mathcal{A}_g$ of Riemann surfaces with automorphisms group $D_{g+1} \times C_2$. For an in nite number of genera this is the only family. Accola-Maclachlan curve is the family.

Joint work with M. Izquierdo
Partially supported by MTM2014-55812

Limit points of the hyperelliptic locus

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Abstract. Let $\mathcal{M}_g$ be the moduli space of compact connected hyperbolic surfaces of genus $g \geq 2$ and let $\mathcal{H}_g \subset \mathcal{M}_g$ be the hyperelliptic locus of $\mathcal{M}_g$, i.e., the subset of hyperbolic surfaces admitting the hyperelliptic involution. Let $\widehat{\mathcal{M}}_g$ be the augmented moduli space, which is a compactification of $\mathcal{M}_g$ by adding *stable hyperbolic surfaces* (i.e., hyperbolic surfaces with nodes). We say that a stable hyperbolic surface $X \in \widehat{\mathcal{M}}_n - \mathcal{M}_n$ is *hyperelliptic* if it is limit of hyperelliptic hyperbolic surfaces, that is, if it is in the boundary of $\mathcal{H}_n$. In [1] it is proved that stable hyperelliptic surfaces share the defining property of hyperelliptic surfaces, that is, a stable hyperbolic surface X is hyperelliptic if and only if it admits an involution σ so that X/σ has genus 0. Moreover, if such an involution exists, it is unique.
In this talk we will describe the combinatorial types of the stable surfaces in $\partial\mathcal{H}_n$. This will be done using the main result in [2].

Referencias

[1] E. Arbarello, M. Cornalba, P.A. Griffiths, **Geometry of Algebraic Curves**, Vol. II, Springer-Verlag Berlin Heidelberg 2011.

[2] Díaz, R. and González-Aguilera, V. **Limit points of the branch locus of $\mathcal{M}_g$**, to appear in Advances in Geometry.

Joint work with Víctor González-Aguilera, UTFSM, Chile

Galois action on solenoids

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Abstract. Let C be a complex algebraic curve uniformised by a Fuchsian group Γ. For any element $\sigma \in G := Gal(\mathbb{C}/\mathbb{Q})$ the natural Galois action of G on defining equations of curves yields a new algebraic curve C^σ with uniformising group Γ^σ.
Little seems to be known about the relationship between Γ and Γ^σ as subgroups of $PSL_2(\mathbb{R})$. By extending the action of G to the solenoid associated to C one can find the following Galois invariants:
i) The arithmeticity or otherwise of the group Γ (thereby providing a proof of this fact independent of Kazhdan's),
ii) The torsion of the commensurator Comm(Γ).
By using ii) I will attempt to present explicit examples of non Galois conjugate (arithmetic) Fuchsian groups.

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On the Connectedness of the Branch Locus in Schottky Space

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Abstract. Schottky space $\mathcal{S}_g$ is the space that parametrizes $\mathrm{PSL}_2(\mathbb{C})$-conjugacy classes of Schottky groups of rank $g \geq 2$. The branch locus $\mathcal{SB}_g$ consists of the conjugacy classes of those Schottky groups which are a finite index proper subgroup of some Kleinian group. We show that $\mathcal{SB}_g$ is always connected.

Joint work with Rubén Hidalgo
Financiado por Fondecyt 1150003, Anillo ACT 1415 PIA-CONICYT and Redes Etapa Inicial Grant 2017-170071

Properties of extremal Riemann surfaces

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Abstract. A Riemann surface shall be called *extremal* if it admits the maximal possible number of symmetries or if it admits the maximal number of ovals for a set of non-conjugate symmetries with ovals. We shall present a variety of recent results concerning such surfaces, in particular we show the structure of the automorphism group and find all the possible topological types of commuting symmetries in an extremal configuration. Special attention shall be paid to the surfaces of even genus and in particular the so-called *button-like* surfaces, being surfaces of genus $g = 4k, k \geq 1$ having both the maximal number of symmetries and the maximal total number of ovals, with a non-abelian automorphism group.

Referencias

[1] E. Kozłowska-Walania, Real equations for Riemann surfaces admitting an extremal configuration of three symmetries, submitted.

[2] G. Gromadzki, E. Kozłowska-Walania, The groups generated by maximal sets of symmetries of Riemann surfaces and extremal quantities of their ovals, *Moscow Math. Journal* 18 (3), (2018), 421-436.

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Riemann surfaces defined over the reals

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Abstract. The known (explicit) examples of Riemann surfaces not definable over their field of moduli are not real but their field of moduli is a subfield of the reals [3, 5, 4, 1]. In this talk we provide explicit families of non-hyperelliptic and hyperelliptic real Riemann surfaces which cannot be defined over their field of moduli [2].

Referencias

[1] M. Artebani, S. Quispe. Fields of moduli and fields of definition of odd signature curves. *Archiv der Mathematik* **99** (2012), 333–343.

[2] E. Badr, R. Hidalgo, S. Quispe. *Riemann surfaces defined over the reals*, Archiv der Mathematik **110** (2018), 351–362.

[3] C. J. Earle. On the moduli of closed Riemann surfaces with symmetries. Advances in the Theory of Riemann Surfaces (1971), 119–130. Ed. L.V. Ahlfors et al. (Princeton Univ. Press, Princeton).

[4] R. A. Hidalgo. Non-hyperelliptic Riemann surfaces with real field of moduli but not definable over the reals. *Archiv der Mathematik* **93** (2009), 219–222.

[5] G. Shimura. On the field of rationality for an abelian variety. *Nagoya Math. J.* **45** (1971), 167–178.

Joint work with E. Badr and R. A. Hidalgo
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On large automorphism groups of compact Riemann surfaces

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Abstract. Belolipetsky and Jones classified those compact Riemann surfaces of genus g admitting a large group of automorphisms of order $\lambda(g-1)$, for each $\lambda > 6$, under the assumption that $g-1$ is a prime number. In this talk the remaining large cases are considered; namely, we classify Riemann surfaces admitting $5(g-1)$ and $6(g-1)$ automorphisms, with $g-1$ a prime number. We also provide isogeny decompositions of their Jacobian varieties.

Joint work with Milagros Izquierdo
Partially supported by Fondecyt Grant 11180024, Redes Etapa Inicial Grant 2017-170071 and Anillo ACT1415 PIA-CONICYT Grant

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Decomposition of Jacobian varieties of curves with dihedral actions via equisymmetric stratification

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Abstract. In this talk we present results on the group algebra decomposition of a Jacobian variety corresponding to a Riemann surface with the action of a finite Group G. We show that it depends on the topological class of the action. More precisely, given a compact Riemann surface X with an action of a finite group G, the group algebra $\mathbb{Q}[G]$ provides an isogenous decomposition of its Jacobian variety JX, known as the group algebra decomposition of JX. We consider the set of equisymmetric Riemann surfaces $\mathcal{M}_{2n-1}^{D_{2n},\theta}$ for all $n \geq 2$. We study the group algebra decomposition of the Jacobian JX of every curve $X \in \mathcal{M}_{2n-1}^{D_{2n},\theta}$ for all admissible actions, and we provide affine models for them. We use the topological equivalence of actions on the curves to obtain facts regarding its Jacobians.

Joint work with Milagros Izquierdo y Leslie Jiménez
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Superelliptic curves with complex multiplication

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Abstract. In this talk we study which algebraic curves over $\mathbb{C}$, or equivalently compact Riemann surfaces, have Jacobian with complex multiplication. The question originates from F. Ort and has been studied by many authors lately. We focus on superelliptic curves and determine conditions on their automorphism group such that their Jacobians have complex multiplication.

Fixed points on asymmetric surfaces

Ewa Tyszkowska

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Abstract. IA Riemann surface of genus $g \geq 2$ is called pseudo-real or asymmetric, if it admits anticonformal automorphisms but none of them is an involution. The orders of such automorphisms are divisible by 4. For a given pair (g, k) of integers greater than 1, we determine the number of classes of asymmetric surfaces in $\mathbb{M}_g$ whose full automorphism group G is isomorphic to $\mathbb{Z}_{4k}$. We study fixed points of conformal automorphisms in G. In particular, we consider the case when all automorphisms of prime orders in G have the same number of fixed points. We construct fixed point free actions on asymmetric surfaces for which a given finite group stands as the group of conformal automorphisms of the surface, and we solve the minimal genus problem in the case when this group is abelian.

Joint work with Ewa Kozłowska-Walania